

1. SOLUTION TO EXERCISE 7 IN ASSIGNMENT 6

Exercise 1.1. Find the Fourier coefficients of the absolute value function $f(t) = |t|$.

Solution. **This is a correction of a mistake I made (the value of a_0) in the recitation class of April 2.**

By definition,

$$(1.1) \quad a_0 = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} |t| dt = 2 \frac{1}{\sqrt{2\pi}} \int_0^{\pi} t dt = \frac{\pi}{\sqrt{2}}.$$

$$(1.2) \quad b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} |t| \sin(kt) dt = 0.$$

(the integration of an odd function over a symmetric domain is zero)

$$(1.3) \quad \begin{aligned} c_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} |t| \cos(kt) dt = \frac{2}{\pi} \int_0^{\pi} t \cos(kt) dt = \frac{2}{k\pi} \left(t \sin(kt) \Big|_0^{\pi} - \int_0^{\pi} \sin(kt) dt \right) \\ &= \frac{2}{k^2\pi} \cos(kt) \Big|_0^{\pi} = \frac{2}{k^2\pi} \left((-1)^k - 1 \right). \end{aligned}$$

REFERENCES

- [1] Otto Bretscher, *Linear Algebra with Application*, 5th ed., Pearson, December 20, 2012.